CHAPTER-1

RELATIONS AND FUNCTIONS

SYNOPSIS :

- **Empty relation** is the relation R in X given by R = $\phi \subset$ X x X.
- **Universal relation** is the relation R in X given R = X x X.
- **Reflexive relation** R in X is a relation with (a, a) $\in$ R = $\forall$ a $\in$ X.
- **Symmetric relation** R in X is a relation satisfying (a, b) $\in$ R implies (b, a) $\in$ R.
- **Transitive relation** R in X is a relation satisfying (a, b) $\in$ R and (b, c) $\in$ R implies that (a, c) $\in$ R.
- **Equivalence relation** R in X is a relation which is reflexive, symmetric and transitive.
- If R is an equivalence relation on a **non-empty set** A then we have :

  (i) a $\in$ [a] for every a $\in$ A

  (ii) [a] = [b] iff a R b

  (iii) For a, b $\in$ A, either [a] = [b] or [a] $\cap$ [b] = ϕ
- A function f : X $\rightarrow$ Y is **one-one (or injective)** if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2 \ \forall \ x_1, x_2 \in X$.

- A function $f : X \to Y$ is **onto (or surjective)** if given any $y \in Y$, $\exists\, x \in X$ such that $f(x) = y$.
- Let A and B be two non-empty sets. A correspondence between the elements of A and B is called a '**function**' from A to B if to each element of A, there corresponds exactly one element of B.
- Let $f : A \to B$ be a function, then :

  (a) A is called the **domain** of 'f'.

  (b) B is called the **codomain** of 'f'.

  (c) The set $\{f(x) : x \in A\}$ is called the range of 'f'. **Range** (f) is a subset of B.
- Two functions f and g defined from A to B are said to be **equal functions** if the images of elements of A under f and g equal. Symbolically, $f, g : A \to B$ are equal if $f(x) = g(x)\ \forall x \in A$.

  $f \neq g \Rightarrow f(x) \neq g(x)$ for at least one $x \in A$.
- Let f and g be two real functions and $X = D(f) \cap D(g)$.

  (i) $(f + g)(x) = f(x) + g(x) \quad \forall x \in X$

  (ii) $(f - g)(x) = f(x) - g(x) \quad \forall x \in X$

  (iii) $(fg)(x) = f(x) \cdot g(x) \quad \forall x \in X$

  (iv) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \forall x \in X - \{x : g(x) = 0\}$

- A function $f : X \to Y$ is **one-one and onto (or bijective)**, if f is both one-one and onto.
- The **composition of functions** $f : A \to B$ and $g : B \to C$ is the function $gof : A \to C$ given by $gof(x) = g(f(x)) \ \forall \ x \in A$.
- A function $f : X \to Y$ is **invertible** if $\exists \ g : Y \to X$ such that $gof = I_X$ and $fog = I_Y$.
- A function $f : X \to Y$ is **invertible** if and only if f is one-one and onto.
- Given a finite set X, a function $f : X \to X$ is one-one (respectively onto) if and only if f is onto (respectively one-one). This is the **characteristic property** of a finite set. This is not true for infinite set.
- If $f : A \to B$ and $g : B \to C$ be functions then the **composite function** $gof : A \to C$ is defined by $(gof)(a) = g(f(a))$, $a \in A$.
- A **binary operation** * on a set A is a function * from A x A to A.

  Let * be a binary operation on A.

  (i) * is called **'commutative'** if $a * b = b * a$ for $a, b \in A$.

  (ii) * is called **'associative'** if $(a * b) * c = a * (b * c)$ for $a, b, c \in a$

  (iii) $e \in A$ is called an **'identity element'** for * if

  $a * e = a = e * a$ for $a \in A$.

  (iv) For $a \in A$, an element $b \in A$ is called an **'inverse'** of a of

  $a * b = e = b * a$.
- The **inverse of the element** a is denoted by a^{-1}.

(i) If * is a binary operation defined on a non-empty set A then the identity element (if exists) is unique.

(ii) If * is an associative binary operation with identity on a non-empty set A then an invertible element of A has unique inverse.